Syllabus
for
Four-Year (Eight-Semester) Undergraduate Program
in
Mathematics (Major)
as per NEP 2020
(Initially this contains Semester-I to Semester-VI Syllabus)

(Effective from Academic Session 2024-25)

University of Gour Banga
Malda-732103
West Bengal

Name of Major (Core) Papers

Semester	Course Code	Course Name
I	MTM-DC-MJ-101	Algebra
II	MTM-DC-MJ-201	Calculus & Analytical Geometry
III	MTM-DC-MJ-301	Abstract Algebra
	MTM-DC-MJ-302	Differential Equations
IV	MTM-DC-MJ-401	Real Analysis I
	MTM-DC-MJ-402	Mechanics
	MTM-DC-MJ-403	Numerical Methods & C Programming Language
V	MTM-DC-MJ-501	Linear Algebra
	MTM-DC-MJ-502	Real Analysis II
	MTM-DC-MJ-503	Multivariate Calculus & Metric Spaces
	MTM-DC-MJ-504	LPP, Game Theory & Integral Transforms I
VI	MTM-DC-MJ-601	Probability, Statistics & Vector Calculus
	MTM-DC-MJ-602	Complex Analysis
	MTM-DC-MJ-603	PDE, Integral Equations & Integral Transforms II
	MTM-DC-MJ-604	Higher Abstract Algebra

Note: Each course is of 4 credits (75 marks, out of which 25 marks is allotted for Continuous Assessment (**CA**) and 50 marks is allotted for Semester-End (**SE**) examination.)

SEMESTER I

MTM-DC-MJ-101

Algebra
Credit: 4
Full Marks: 75(CA: 25, SE: 50)

Learning Objectives:
The principal aim of this course is to introduce the concepts of complex numbers and to present a systemic introduction to theory of equations, number theory and basic course on algebra, linear algebra and its applications.

Learning Outcomes:
On completion of the course, the students would

1. familiarize with the concepts of complex numbers and gain knowledge about its various properties and its applications to solve various problems.
2. gain knowledge about a basic introduction to algebra which has different applicability in various branches of science.
3. be introduced with the basic concepts of number theory and its applications in advanced mathematics and in various practical fields e.g., cryptography, computer science.
4. gain knowledge about system of linear equations and learn various methods to solve them.
5. demonstrate the knowledge of characteristic polynomials, eigenvalue, eigenvectors to solve various problems.
6. familiarize with various methods to find roots of a polynomial equation and apply them to practice.
7. apply the concepts of eigenvalue, eigenvectors to verify the diagonalization of a square of matrix. gain idea about real quadratic forms of a matrix and solve related problems.

Course Contents

Module-1
Polar representation of complex numbers, n-th roots of unity, De Moivre's theorem for rational indices and its applications. Inequality: Inequalities involving $AM \geq GM \geq HM$, m-th power theorem, Cauchy-Schwartz inequality, Maximum and minimum values of polynomials.

Module-2
General properties of equations, Fundamental theorem of classical algebra (statement only) and

its application on exponential, sine, cosine and logarithm of a complex number, Transformation of equations, Descarte's rule of signs for positive and negative rule, Strum's theorem, Relation between the roots and the coefficients of equations, Symmetric functions, Applications of symmetric function of the roots, Solutions of reciprocal and binomial equations, Algebraic solutions of the cubic (Cardon's method) and biquadratic (Ferrari's method).

Module-3

Functions, Composition of functions, Invertible functions, One to one correspondence and cardinality of a set. Recurrence relation, definition, example, Formation of recurrence relation, Factorial representation, Fibonacci number, Solution upto second order linear recurrence relation, Generating function. Equivalence relations and partitions.

Well-ordering property of positive integers, Division algorithm, Divisibility and Euclidean algorithm, Congruence relation between integers, Principles of Mathematical Induction, statement of Fundamental Theorem of Arithmetic.

Module-4

Systems of linear equations, row reduction and echelon forms, the matrix equation $Ax = b$, solution sets of linear systems, applications of linear systems, linear independence. Real Quadratic Form involving not more than three variables, Characteristic equation of square matrix of order not more than three, determination of eigenvalues and eigenvectors, Cayley-Hamilton Theorem.

Reference Books

1. T. Andreescu and D. Andrica, Complex Numbers from A to . . . Z, Birkhauser Boston, 2008.
2. D.C. Lay, S.R. Lay and J.J. McDonald, Linear Algebra and its Applications, 5rd Ed., Pearson, 2014.
3. K.B. Dutta, Matrix and linear algebra, Prentice Hall, 2004.
4. K. Hoffman and R. Kunze, Linear algebra, Prentice Hall, 1971.
5. W.S. Burnstine and A.W. Panton, Theory of equations, Nabu Press, 2011.
6. S.H. Friedberg, A.J. Insel and L.E. Spence, Linear Algebra, 4th Ed., PHI, 2004.
7. S. Bernard and J.M. Child, Higher Algebra, Macmillan and Co. 1952.

SEMESTER II

MTM-DC-MJ-201

Calculus & Analytical Geometry
Credit: 4
Full Marks: 75 (CA: 25, SE: 50)

Learning Objectives:
The primary objective of this course is to study calculus and its applications in various fields and understand the basic knowledge of geometry in two and three dimensional spaces.

Learning Outcomes:
On completion of the course, the students would

1. gain knowledge about higher order derivatives and its applications, concavity of curves, asymptotes, and curve tracing techniques.
2. be able to parametrize curves, sketch functions and plot them.
3. gain knowledge about reduction formula for integration of functions like $\sin nx$, $\sin^m x$, $\sin^n x$ etc., area of surface of revolution, parametric curves etc.
4. familiarize with the concepts of limit, continuity, and differentiability of functions of single variable and apply them to solve various problems.
5. be able to apply the knowledge of geometry in 2D to find the angle between two straight lines, to find the equation of bisectors, equation of tangent and normal.
6. gain knowledge about classification of conics and conicoid, polar equation of conics.
7. able to visualize standard quadratic surfaces like sphere, cone, ellipsoid etc.

Course Contents

Module-1
Real-valued functions defined on an interval, limit of a function (Cauchy's definition), Algebra of limits, Continuity of a function at a point and in an interval, Properties of continuous functions (statement only) and its related problems on closed intervals, Hyperbolic functions. Higher order derivatives, Leibnitz rule of successive differentiation and its applications to problems of type $e^{ax+b}\sin x$, $e^{ax+b}\cos x$, $(ax+b)^n\sin x$, $(ax+b)^n\cos x$, concavity, convexity and points of inflection, envelopes, asymptotes, radius of curvature.

Module-2
Reduction formulae, derivations and illustrations of reduction formulae of the type integration of

$\sin^n x$, $\cos^n x$, $\tan^n x$, $\sec^n x$, $(\log x)^n$, $\sin^n x \sin^m x$, evaluation of definite integrals, integration as the limit of a sum, concept of improper integration, use of Beta and Gamma functions, parametric equations, parametrizing a curve, arc length, arc length of parametric curves, area of surface of revolution, volume enclosed by closed surface of revolution.

Module-3

Reflection properties of conics, translation and rotation of axes and second degree equations, reduction and classification of conics using the discriminant, Point of intersection of two intersecting straight lines. Angle between two lines, Equation of bisectors. Equation of two lines joining the origin to the points in which a line meets a conic. Equations of pair of tangents from an external point, chord of contact, Polar equations of straight lines and conics. Equation of chord joining two points. Equations of tangent and normal, Poles and Polars.

Module-4

Spheres, Cylindrical surfaces, Central conicoids, paraboloids, plane sections of conicoids, Generating lines, Classification of quadrics, illustrations of quadric surfaces, like cone, cylinder, paraboloid, ellipsoid, hyperboloid.

Reference Books

1. S.L. Loney, The Elements of Coordinate Geometry, Macmillan and Co., 1895.
2. G.B. Thomas and R.L. Finney, Calculus, 9th Ed., Pearson, 2005.
3. M.J. Strauss, G.L. Bradley and K.J. Smith, Calculus, 3rd Ed., Pearson Education, 2007.
4. H. Anton, I. Bivens and S. Davis, Calculus, 10th Ed., John Wiley and Sons Inc., 2012.
5. R. Courant and F. John, Introduction to Calculus and Analysis (Volumes I & II), Springer, 1989.
6. T.M. Apostol, Calculus (Volumes I & II), John Wiley & Sons, 1967.
7. S. Goldberg, Calculus and mathematical analysis.
8. S. Lang, A First Course in Calculus, Springer 1998.
9. K.A. Ross, Elementary Analysis: The Theory of Calculus, Springer, 2nd ed., 2013.
10. R.J.T. Bell, An Elementary Treatise on Coordinate Geometry of Three Dimensions, Macmillan Publishers India Limited, 2000.

SEMESTER III

MTM-DC-MJ-301

Abstract Algebra
Credit: 4
Full Marks: 75 (CA: 25, SE: 50)

Learning Objectives:
The primary objective of this course is to introduce the fundamentals of several algebraic structures including groups, rings, integral domains and fields. This course is intended to develop the students' ability to handle abstract ideas of Mathematics and Mathematical proofs.

Learning Outcomes:
On completion of this course, the students will be able to:

1. Acquire knowledge of important mathematical concepts in abstract algebra such as definition of a group, order of a finite group, order of an element in a group, subgroups, centralizer, normalizer, centre of a group, cyclic groups, permutation groups, etc.
2. Familiar with cosets, normal subgroups, quotient groups, group homomorphisms, isomorphisms and the isomorphism theorems.
3. Understand the concepts of rings, subrings, integral domains and fields, characteristic of a ring and ring homomorphisms.
4. Comprehend ideals including prime and maximal ideals, ring homomorphisms, isomorphisms, isomorphism theorems, fields of quotients, polynomial rings, etc.

Course Contents

Module-1
Definition and examples of groups, elementary properties of groups. Subgroups and examples of subgroups, centralizer, normalizer, centre of a group. Properties of cyclic groups, classification of subgroups of cyclic groups. Permutation group, cycle notation for permutations, properties of permutations, even and odd permutations, alternating group.

Module-2
Cosets, properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem. Normal subgroup and quotient group. Group homomorphisms, properties of homomorphisms, properties of isomorphisms. First isomorphism theorem. Second isomorphism theorem and Third isomorphism theorem (Statement only).

Module-3

Definition and examples of rings, elementary properties of rings, subrings, integral domains and fields, Elementary properties of fields, sub-fields, characteristic of a ring. Ring homomorphisms, properties of ring homomorphisms.

Module-4
Ideal, Prime and Maximal Ideals, First Isomorphism theorem for ring. Isomorphism theorems II and III for ring (Statement and Applications), field of quotients. Introduction to polynomial ring.

Reference Books

1. J.B. Fraleigh, A First Course in Abstract Algebra, 7th Ed., Pearson, 2002.
2. M. Artin, Abstract Algebra, 2nd Ed., Pearson, 2011.
3. J.A. Gallian, Contemporary Abstract Algebra, 8th Ed., Houghton Mifflin, 2012.
4. J.J. Rotman, An Introduction to the Theory of Groups, 4th Ed., Springer, 1995.
5. I.N. Herstein, Topics in Algebra, Wiley Eastern Limited, 1975.
6. D.S. Malik, J.M. Mordeson and M.K. Sen, Fundamentals of Abstract Algebra, McGraw Hill, 1996.
7. D.S. Dummit and R.M. Foote, Fundamentals of Abstract Algebra, 3rd Ed., Wiley, 2003.
8. M.K. Sen, S. Ghosh, P. Mukhopadhyay and S.K. Maiti, Topics in Abstract Algebra, 3rd Ed., Universities Press, 2019.

MTM-DC-MJ-302

Differential Equations
Credit: 4
Full Marks: 75 (CA: 25, SE: 50)

Learning Objectives:
The fundamental goal of this course is to study ordinary and partial differential equations through analytic and qualitative approaches and its applications in various physical problems.

Learning Outcomes:
On completion of the course, the students would

1. gain knowledge about qualitative analysis of the ordinary differential equations.
2. get idea of ordinary and partial differential equations, linear and non-linear equations, concepts of order and degree.
3. be able to demonstrate the knowledge of Existence and Uniqueness theory for Initial Value Problems in ordinary differential equations.

4. be able to apply the knowledge of ordinary differential equations to solve problems in different areas of applied mathematics.

5. Learn about the various techniques to solve ordinary differential equations and apply them to solve various physical problems.

6. gain knowledge about the distinctive features of various types of ordinary differential equations.

7. be familiarize with the concepts of equilibrium points and study stability analysis, interpretation of phase plane.

8. learn about solution techniques to solve partial differential equations by Lagrange's and Charpit's methods and apply them to solve various problems.

9. be introduced with the concepts of canonical forms of first order linear partial differential equations.

Course Contents

Module-1
Exact, linear and Bernoulli's equations, trajectories. Equations not of first degree, Clairaut's equations, singular solution. Existence & Uniqueness theorem for 1st order IVP Picard's Theorem (Statement only). General solution of homogeneous equation of second order, principle of super position for homogeneous equation, Wronskian and its properties. Linear homogeneous and non-homogeneous equations of higher order with constant coefficients, Cauchy-Euler equation, method of undetermined coefficients, method of variation of parameters, Eigenvalue problem.

Module-2
Systems of linear differential equations, types of linear systems, differential operators, an operator method for linear systems with constant coefficients, Basic theory of linear systems in normal form, homogeneous linear systems with constant coefficients: Two equations in two unknown functions. Equilibrium points, Interpretation of the phase plane.

Module-3
Power series solution of a differential equation about an ordinary point, solution about a regular singular point. Legendre polynomials, Bessel functions of the first kind and their properties.

Module-4
Partial differential equations, basic concepts and definitions. First-order equations: classification, construction and geometrical interpretation. Method of characteristics for obtaining general solution of quasi linear equations. Canonical forms of first-order linear equations. Solution by Lagrange's and Charpit's method.

Reference Books

1. G.F. Simmons, Differential Equations with Applications and Historical Notes, McGraw Hill, 2017.

2. S.L. Ross, Differential Equations, 3rd Ed., Wiley, 2007.

3. C.H. Edwards and D.E. Penny, Differential Equations and Boundary Value Problems Computing and Modeling, Pearson, 2005.

4. M.L. Abel and J.P. Braselton, Differential Equations with MATHEMATICA, 3rd Ed., Elsevier, 2004.

5. D. Murray, Introductory Course in Differential Equations, Orient Longman, 2003.

6. W.E. Boyce and R.C. Diprima, Elementary Differential Equations and Boundary Value Problems, Wiley, 2009.

7. E.A. Coddington, An Introduction to Ordinary Differential Equations, Dover Publications Inc., 1989.

8. H.T.H. Piaggio, Elementary Treaties on Differential Equations and their Applications, G. Bell and Sons, 1920.

9. I.N. Sneddon, Elements of Partial Differential equations, McGraw-Hill, 1957.

10. T. Myint-U and L. Debnath, Linear Partial Differential Equations for Scientists and Engineers, 4th Ed., Springer, 2006.

11. F.H. Miller, Partial Differential Equations, John Wiley & Sons, 1941.

SEMESTER IV

MTM-DC-MJ-401

Real Analysis I
Credit: 4
Full Marks: 75 (CA:25 , SE: 50)

Learning Objectives:

The main objective of this course is to comprehend theoretical knowledge and have practical skills in the subject of real analysis and to demonstrate an ability to initiate and sustain in-depth study relevant to real analysis.

Learning Outcomes:

On completion of this course, the students will be able to:

1. Demonstrate competence with the algebraic and order properties of real numbers, describe the real line as a complete ordered field, and understand the topology of the real line.
2. Acquire knowledge of elementary properties of real sequences by finding limits and proving results involving sum, difference, product and quotients of sequences, and apply several tests to examine convergence of real series.
3. Understand the concepts of limits, continuity and uniform continuity of real-valued functions, and analyse their several properties.
4. Demonstrate differentiability of real-valued functions and its various properties, apply Rolle's theorem, Lagrange's mean value theorem, Cauchy's mean value theorem to solve assorted problems in the context of real analysis as well as calculus.

Course Contents

Module-1

Development of real numbers. The algebraic properties of $\mathbb{R}$, rational and irrational numbers, the order properties of $\mathbb{R}$. Absolute value and the real line, bounded and unbounded sets in $\mathbb{R}$, supremum and infimum, neighbourhood of a point. The completeness property of $\mathbb{R}$, the Archimedean property, density of rational numbers in $\mathbb{R}$, nested intervals property, binary representation of real numbers, uncountability of $\mathbb{R}$. Closed set, open set, closure and interior of a subset of the real line.

Module-2

Sequences, the limit of a sequence and the notion of convergence, bounded sequences, limit theorems, squeeze theorem, monotone sequences, monotone convergence theorem. Subsequences, monotone subsequence theorem and the Bolzano-Weierstrass theorem, the divergence criterion, limit superior and limit inferior of a sequence, Cauchy sequences, Cauchy's convergence criterion. Infinite series, convergence and divergence of infinite series. Tests for Convergence. Alternating series, absolute and conditional convergence.

Module-3
Sequential criterion for limits, divergence criteria. Limit theorems, infinite limits and limits at infinity. Continuous functions, sequential criterion for continuity and discontinuity. Algebra of continuous functions. Continuous functions on an interval and its properties, intermediate value theorem, location of roots theorem, preservation of intervals theorem. Uniform continuity, non-uniform continuity criteria, uniform continuity theorems.

Module-4
Differentiability of a function at a point and in an interval, Caratheodory's theorem, chain rule, derivative of inverse functions, algebra of differentiable functions. Mean value theorems: Rolle's Theorem, Lagrange's mean value theorem, Cauchy's mean value theorem. Applications of mean value theorem to inequalities, relative extremum. The intermediate value property of derivatives, Darboux's theorem. L'Hospital's rule. Taylor's theorem and its application. Expansion of functions.

Reference Books

1. R.G. Bartle and D.R. Sherbert, Introduction to Real Analysis, 3rd Ed., Wiley, 2000.
2. G.G. Bilodeau , P.R. Thie and G.E. Keough, An Introduction to Analysis, 2nd Ed., Jones & Bartlett, 2009.
3. B.S. Thomson, A.M. Bruckner and J.B. Bruckner, Elementary Real Analysis, Prentice Hall, 2001.
4. S.K. Berberian, A First Course in Real Analysis, Springer, 1998.
5. T.M. Apostol, Mathematical Analysis, Narosa, 2002.
6. R. Courant and F. John, Introduction to Calculus and Analysis, Vol I, Springer, 1999.
7. W. Rudin, Principles of Mathematical Analysis, McGraw Hill, 2017.
8. C.C. Pugh, Real Mathematical Analysis, Springer, 2002.
9. T. Tao, Analysis I, Hindustan Book Agency, 2006
10. S. Goldberg, Calculus and mathematical analysis.
11. H.R. Beyer, Calculus and Analysis, Wiley, 2010.
12. S. Lang, Undergraduate Analysis, Springer, 2nd Ed., 1997.
13. A. Kumar and S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.

MTM-DC-MJ-402

Mechanics
Credit: 4
Full Marks: 75 (IA: 25, SE: 50)

Learning Objectives:

The objective of this course is to build up problems based mathematical skills in the areas of mechanics and apply these skills to the solution of a variety of practical problems appearing in physical sciences.

Learning Outcomes:

On completion of this course, the students will be able to

1. Understand concepts of kinematics of a particle, Newton laws of motion, law of gravitation and solve related problems.
2. Solve a variety of problems based on Work, power, kinetic energy, conservative forces-potential energy, energy conservation in a conservative field, Stable equilibrium and small oscillations and impulsive forces.
3. Solve assorted problems in particle dynamics including rectilinear motion, simple harmonic motion, oscillations, motion of elastic strings and springs, etc.
4. Work out problems on planar motion of a particle, orbits in a central force field, motion under the attractive inverse square law, Kepler's laws on planetary motion, etc.

Course Contents

Module-1

Kinematics of a particle: Velocity, acceleration, angular velocity, linear and angular momentum. Relative velocity and acceleration. Expressions for velocity and acceleration in case of rectilinear motion and planar motion in Cartesian and polar coordinates, tangential and normal components. Uniform circular motion, radial and cross radial components of velocity and acceleration.

Newton laws of motion and law of gravitation: Space, time, mass, force, inertial reference frame, principle of equivalence and g. Vector equation of motion.

Module-2

Work, power, kinetic energy, conservative forces-potential energy. Existence of potential energy function.

Energy conservation in a conservative field. Stable equilibrium and small oscillations: Approximate equation of motion for small oscillation. Impulsive forces

Module-3

Problems in particle dynamics: Rectilinear motion in a given force field - vertical motion under uniform gravity, inverse square field, constrained rectilinear motion, vertical motion under gravity in a resisting medium, simple harmonic motion, Damped and forced oscillations, resonance of an oscillating system, motion of elastic strings and springs.

Module-4

Planar motion of a particle: Motion of a projectile in a resisting medium under gravity, orbits in a central force field, Stability of nearly circular orbits. Motion under the attractive inverse square law, Kepler's laws on planetary motion. Slightly disturbed orbits, motion of artificial satellites. Constrained motion of a particle on smooth and rough curves. Equations of motion referred to a set of rotating axes.

Reference Books

1. R.D. Gregory, Classical Mechanics, Cambridge University Press, 2006.
2. K.R. Symon, Mechanics, Addison Wesley, 1971.
3. M. Lunn, A First Course in Mechanics, Oxford University Press, 1991.
4. J.L. Synge and B.A. Griffith, Principles of Mechanics, Mcgraw Hill, 1949.
5. T.W.B. Kibble, F.H. Berkshire, Classical Mechanics, Imperial College Press, 2004.
6. D.T. Greenwood, Principle of Dynamics, Prentice Hall, 1987.
7. F. Chorlton, Textbook of Dynamics, E. Horwood, 1983.
8. D. Kleppner and R. Kolenkow, Introduction to Mechanics, Mcgraw Hill, 2017.
9. A.P. French, Newtonian Mechanics, Viva Books, 2011.
10. S.P. Timoshenko and D.H. Young, Engineering Mechanics, Schaum Outline Series, 4th Ed., 1964.
11. D. Chernilevski, E. Lavrova and V. Romanov, Mechanics for Engineers, MIR Publishers, 1984.
12. I.H. Shames and G.K.M. Rao, Engineering Mechanics: Statics and Dynamics, 4th Ed., Pearson, 2009.
13. R.C. Hibbeler, Engineering Mechanics: Statics and Dynamics, 11th Ed., Pearson, 2011.
14. S.L. Loney, An Elementary Treatise on the Dynamics of Particle and of Rigid Bodies, Cambridge University Press, 2017.
15. S.L. Loney, An Elementary Treatise on Statics, Cambridge University Press, 2016.
16. R.S. Verma, A Textbook on Statics, Pothishala, 1962.
17. A.S. Ramsey, Dynamics (Part I & II), Cambridge University Press, 1952.

MTM-DC-MJ-403

Numerical Methods & C Programming Language

Credit: 4

Full Marks: 75 (CA: 25, SE: 50)

Learning Objectives:

The objective of this course to demonstrate understanding of common numerical methods and how they are used to obtain approximate solutions to otherwise intractable mathematical problems, together with and overview of the C-Programming languages.

Learning Outcomes:

On completion of this course, the students will be able to

1. Understand the importance of error analysis and their propagation, and familiar with the methods of solving algebraic and transcendental equations.
2. Learn the methods of solving system of linear and non-linear equations, and understand the techniques of interpolation and numerical differentiation.
3. Acquire knowledge of numerical integration, algebraic eigenvalue problem and initial value problems
4. Develop a C program, control its sequence, and produce logical outputs.

Course Contents

Module-1

Errors: Relative, Absolute, Round off, Truncation. Transcendental and Polynomial equations: Bisection method, Secant method, Regula-Falsi method, fixed point iteration, Newton-Raphson method. Convergence of these methods.

Module-2

System of linear algebraic equations: Gaussian elimination, Gauss Jordan, LU decomposition methods. Gauss Jacobi method, Gauss Seidel method and their convergence analysis.

Finite difference operators, Divided difference. Interpolation: Newton's and Lagrange methods. Error bounds. Central difference interpolation. Numerical differentiation. Approximation: Least square polynomial approximation.

Module-3

Numerical Integration: Newton Cotes formula, Trapezoidal rule, Simpson's 1/3rd rule, Simpsons 3/8th rule, Weddle's rule, Composite Trapezoidal rule, Composite Simpson's 1/3rd rule. Gauss quadrature formula: Gauss-Legendre, Gauss-Chebyshev, Gauss-Laguerre and Gauss-Hermite quadrature formula.

The algebraic eigenvalue problem: Power method.

Initial Value Problems: The method of successive approximations, Euler's method, the modified Euler method, Runge-Kutta methods of orders two and four.

Module-4

Overview of the C-Programming Languages, Data Type, Constants and Variables, Input and Output, Operators and Expressions, if-else Statement, switch Statement, for Loop, while Loop, do-while Loop, break and continue, functions, array and simple problems.

Reference Books

1. K.E. Atkinson, An Introduction to Numerical Analysis, John Wiley and Sons, 1978.
2. B. Bradie, A Friendly Introduction to Numerical Analysis, Pearson Education, India, 2007.
3. M.K. Jain, S.R.K. Iyengar and R.K. Jain, Numerical Methods for Scientific and Engineering Computation, 6th Ed., New age International Publisher, 2007.
4. C.F. Gerald and P.O. Wheatley, Applied Numerical Analysis, Pearson Education, India, 2008.
5. U.M. Ascher and C. Greif, A First Course in Numerical Methods, 7th Ed., PHI Learning Private Limited, 2013.
6. John H. Mathews and Kurtis D. Fink, Numerical Methods using Matlab, 4th Ed., PHI Learning Private Limited, 2012.
7. J.B. Scarborough, Numerical Mathematical Analysis, Oxford and IBH, 2005.

SEMESTER V

MTM-DC-MJ-501

Linear Algebra
Credit: 4
Full Marks: 75(CA: 25, SE: 50)

Learning Objectives:
The principal aim of this course is to provide students with a foundational understanding of core linear algebra concepts, including vectors, matrices, systems of linear equations, vector spaces, inner product spaces, linear transformations, determinants, eigenvalues and eigenvectors.

Learning Outcomes:
On completion of the course, the students would

1. Perform operations on matrices and vectors, including addition, scalar multiplication, matrix multiplication, transposition and understand their properties.
2. Determine linear independence, span, basis and dimension of vector spaces/subspaces.
3. Analyze linear transformations, find their matrix representations, and identify their kernel and range.
4. Compute determinants and understand their role in invertibility and solving linear systems.
5. Find eigenvalues and eigenvectors of a linear operator and use them for diagonalization and understanding system dynamics.
6. Understand concepts of inner product spaces, orthogonality, and orthonormal bases (e.g., using the Gram-Schmidt process).

Course Contents

Module-1
Vector spaces over a field, subspaces. Sum and direct sum of subspaces. Quotient spaces, Linear span. Linear dependence and independence. Basis and dimension. Finite dimensional spaces. Existence theorem for bases in the finite dimensional case. Invariance of the number of vectors in a basis, dimension. Existence of complementary subspace of any subspace of a finite dimensional vector space. Dimensions of sums of subspaces. Quotient space and its dimension. Infinite dimensional vector spaces.

Module-2
Linear transformations, null space, range, rank and nullity of a linear transformation, matrix representation of a linear transformation, algebra of linear transformations. Isomorphisms. Isomorphism

theorems, invertibility and isomorphisms, change of coordinate matrix.

Module-3
Linear operators and their eigenvalues and eigenvectors, characteristic equation, eigenspaces, algebraic and geometric multiplicity of eigenvalues. Diagonalization, conditions for diagonalizability. Invariant subspaces and Cayley-Hamilton theorem. Jordan Canonical forms.

Module-4
Inner product spaces. Cauchy-Schwarz inequality. Orthogonal vectors and orthogonal complements. Orthonormal sets and bases. Bessel's inequality. Gram-Schmidt orthogonalization process. Hermitian, Self-adjoint, Unitary and Orthogonal transformation for complex and real spaces. Bilinear and Quadratic forms, real quadratic forms.

Reference Books

1. S.H. Friedberg, A.J. Insel and L.E. Spence, Linear Algebra, 4th Ed., New Delhi: Prentice Hall of India, 2004.
2. I.N. Herstein, Topics in Algebra, John Wiley & Sons, 2006.
3. M. Artin, Algebra, 2nd Ed., Pearson, 2011.
4. A.R. Rao and P. Bhimasankaram, Linear Algebra, Hindustan Book Agency, 2000.
5. S. Roman, Advanced Linear Algebra, Springer, 2008.
6. S. Lang, Introduction to Linear Algebra, 2nd Ed., Springer, 2005.
7. S.D. Dummit and M.R. Foote, Abstract Algebra, Wiley, 2011.
8. J. Hefferon, Linear Algebra, Orthogonal Publishing L3C, 2020.
9. D.C. Lay, S.R. Lay and J. McDonald, Linear Algebra and its Applications, Pearson, 2016.
10. G. Strang, Linear Algebra and its Applications, Thomson, 2007.
11. S. Kumaresan, Linear Algebra: A Geometric Approach, Prentice Hall of India, 1999.
12. K. Hoffman and R.A. Kunze, Linear Algebra, 2nd Ed., Prentice Hall of India, 1971.
13. S. Axler, Linear Algebra Done Right, Springer, 2014.
14. S.J. Leon, Linear Algebra with Applications, Pearson, 2015.
15. J.S. Golan, Foundations of Linear Algebra, Springer, 1995.
16. J.S. Golan, The Linear Algebra a Beginning Graduate Student Ought to Know, Springer, 2012.

MTM-DC-MJ-502

Real Analysis II
Credit: 4
Full Marks: 75 (CA: 25, SE: 50)

Learning Objectives:
The principal aim of this course is to provide students with a rigorous theoretical understanding of monotone functions, functions of bounded variation, Riemann integration, Fourier series, improper integrals, and sequences and series of functions.

Learning Outcomes:
On completion of the course, the students would

1. Analyze and characterize monotone functions and functions of bounded variation, including their properties and the concept of total variation.
2. Understand Riemann integrability of functions, compute Riemann integrals using definitions, and apply fundamental theorems of integral calculus, including change of variables.
3. Derive and analyze Fourier series for periodic functions, understand conditions for convergence (*e.g.*, Dirichlet's condition), and apply Bessel's and Parseval's inequalities.
4. Determine the convergence of improper integrals using various tests (comparison test, M-test, Abel's test, Dirichlet's test) and gain working knowledge of Beta and Gamma functions and their interrelations.
5. Distinguish between pointwise and uniform convergence of sequences and series of functions, and apply theorems regarding the continuity, differentiability, and integrability of limit and sum functions.
6. Apply the Cauchy criterion for uniform convergence and the Weierstrass M-Test to establish the uniform convergence of sequences and series of functions.

Course Contents

Module-1
Properties of monotone functions. Functions of bounded variation, total variation, continuous functions of bounded variation. Curves and paths in $\mathbb{R}^n$ ($n \leq 3$), rectifiable paths and arc length.

Module-2
Riemann integration: upper and lower sums, upper and lower integral, definition and conditions of integrability. Riemann integrability of monotone and continuous functions, elementary properties of the Riemann integral. Intermediate value theorems for integrals. Fundamental theorem of integral calculus. Mean value theorems of integral calculus. Change of variables.

Module-3
Periodic functions, Fourier coefficients and Fourier series, convergence, Bessel's inequality, Parseval's identity, Dirichlet's condition, examples of Fourier series. Improper integrals: Range of

integration, finite or infinite. Necessary and sufficient condition for convergence of improper integral.
Comparison and M-test. Absolute and non-absolute convergence and inter-relations. Statement of Abel's and Dirichlet's test for convergence on the integral of a product. Convergence of Beta and Gamma functions and their inter-relation.

Module-4
Sequence of functions: Pointwise and uniform convergence, Weierstrass M-test, Theorems on continuity, differentiability, integrability and boundedness of the limit function of a sequence of functions. Series of functions: Pointwise and uniform convergence, Theorems on the boundedness, continuity, integrability and differentiability of the sum function of a series of functions; Cauchy criterion for uniform convergence, Weierstrass M-test. Dini's theorem.

Reference Books

1. R. Bartle and D.R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, 2003.
2. K.A. Ross, Elementary Analysis: The Theory of Calculus, Springer, 2004.
3. T.M. Apostol, Mathematical Analysis, Narosa Publishing House, 2002.
4. W. Rudin, Principles of Mathematical Analysis, McGraw Hill, 2017.
5. A. Kumar and S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
6. H.H. Sohrab, Basic Real Analysis, Birkhäuser, 2014.
7. A. Mattuck, Introduction to Analysis, Prentice Hall, 1999.
8. S.R. Ghorpade and B.V. Limaye, A Course in Calculus and Real Analysis, Springer, 2006.
9. R. Courant and F. John, Introduction to Calculus and Analysis, Vol II, Springer, 2000.
10. S.C. Malik and S. Arora, Mathematical analysis, New Age International Pvt. Ltd., 2017.
11. H.L. Royden, Real Analysis, Macmillan Publishing House, 1988.
12. B.K. Lahiri and K.C. Roy, Real Analysis, World Press, 1988.
13. T. Tao, Analysis I, 4th Ed., Hindustan Book Agency, 2022.
14. T. Tao, Analysis II, Hindustan Book Agency, 2006.
15. S. Shirali and H.L. Vasudeva, Metric Spaces, Springer, 2006.
16. G.G. Bilodeau , P.R. Thie and G.E. Keough, An Introduction to Analysis, 2nd Ed., Jones & Bartlett, 2010.
17. B.S. Thomson, A.M. Bruckner and J.B. Bruckner, Elementary Real Analysis, Prentice Hall, 2001.
18. C.C. Pugh, Real Mathematical Analysis, Springer, 2002.
19. H.R. Beyer, Calculus and Analysis, Wiley, 2010.
20. S.K. Berberian, A First Course in Real Analysis, Springer Verlag, New York, 1994.
21. R.R. Goldberg, Methods of Real Analysis, Oxford & IBH Publishing, 1970.

22. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill Education, 2004.
23. S. Lang, Undergraduate Analysis, 2nd Ed., Springer, 1997.
24. S. Abbott, Understanding Analysis, Springer, 2015.
25. S.K. Mapa, Introduction to Real Analysis, Leveant, 2022.

MTM-DC-MJ-503

Multivariate Calculus & Metric Spaces
Credit: 4
Full Marks: 75 (CA: 25, SE: 50)

Learning Objectives:
The principal aim of this course is to equip students with a comprehensive understanding of multivariate calculus, functions of several variables, multidimensional integration, and metric spaces and solve complex problems in scientific and engineering applications.

Learning Outcomes:
On completion of the course, the students would

1. Analyze and apply concepts of limits, continuity, partial derivatives and integration for functions of several variables.
2. Evaluate total derivative, compute Jacobians, and utilize the chain rule for multivariate functions.
3. Locate and classify extrema of functions of several variables, including solving constrained optimization problems using the method of Lagrange multipliers and able to apply Green's theorem, Stokes' theorem & Gauss' divergence theorem in various problems.
4. Able to define a metric space and its fundamental topological elements, including open and closed sets, neighborhoods, interior, closure,and limit points and able to identify and analyze key properties of subsets and spaces, such as density, separability, and the characteristics of subspaces.
5. Analyze convergent and Cauchy sequences, understand complete metric spaces, and apply Cantor's theorem.
6. Define and characterize continuous and uniformly continuous mappings and identify connected and compact metric spaces.

Course Contents

Module-1

Scalar fields, vector fields, limit and continuity, partial derivatives and directional derivatives, Euler's theorem for homogeneous function, Schwarz's and Young's theorems, total derivative and Jacobian, sufficient condition for differentiability, Chain rule, Mean value theorem, Taylor's theorem, Inverse function theorem, Application of Implicit function theorem, gradient, tangent planes. Extrema of functions, method of Lagrange multipliers, constrained optimization problems.

Module-2

Multiple integral: Double integration over rectangular/non-rectangular regions, changing the order of integration. Triple integrals, Triple integral over a parallelepiped and solid regions. Volume by triple integrals. Polar, cylindrical and spherical co-ordinates. Change of variables in double and triple integrals. Concept of line and surface integrals, Green's theorem, Stokes' theorem & Gauss' divergence theorem (Statements and Problems).

Module-3

Metric spaces: Definition and examples. Open and closed balls, neighbourhood, Open sets, Interior of a set. Limit point of a set, Closed sets, closure, subspaces, dense sets, separable spaces. Sequences and their convergence in metric spaces, Cauchy sequences. Complete metric spaces, Cantor's theorem.

Module-4

Continuous mappings, sequential criterion and other characterizations of continuity. Uniform continuity. Connectedness and compactness of metric spaces. Contraction mapping and Banach's fixed point theorem.

Reference Books

1. G.B. Thomas and R.L. Finney, Calculus, 9th Ed., Pearson, 2005.
2. T. Apostol, Calculus, Vol. I and II, Wiley, 2007.
3. P. Lax and M.S. Terrell, Calculus with Applications, Springer, 2014.
4. M.J. Strauss, G.L. Bradley and K.J. Smith, Calculus, 3rd Ed., Pearson, 2007.
5. M. Spivak, Calculus on Manifolds, Addison-Wesley, 1995.
6. J.R. Munkres, Analysis on Manifolds, Addison-Wesley, 1991.
7. C.H. Edwards, Advanced Calculus of Several Variables, Academic Press, 1973.
8. J.J. Callahan, Advanced Calculus a Geometric View, Springer, 2010.
9. J.E. Marsden, A.J. Tromba and A. Weinstein, Basic Multivariable Calculus, Springer, 2005.
10. J. Stewart, Multivariable Calculus: Concepts and Contexts, 4th Ed., Cengage Learning, 2009.
11. T.M. Apostol, Mathematical Analysis, Narosa, 2002.
12. S.R. Ghorpade and B.V. Limaye, A Course in Multivariable Calculus and Analysis, Springer, 2010.
13. R. Courant and F. John, Introduction to Calculus and Analysis, Vol. II, Springer, 1999.
14. W. Rudin, Principles of Mathematical Analysis, McGraw Hill, 2017.
15. T. Shinfrin, Multivariable Mathematics, Wiley, 2005.

16. K.D. Joshi, Multivariate Calculus Through Linear Algebra, New Age International Pvt. Ltd., 2021.

17. D. Somasundaran and B. Choudhary, A First Course in Mathematical Analysis, Narosa, 1996.

18. S. Shirali and H.L. Vasudeva, Metric Spaces, Springer Verlag, London, 2006.

19. S. Kumaresan, Topology of Metric Spaces, 2nd Ed., Narosa Publishing House, 2011.

20. M.Ó. Searcóid, Metric Spaces, Springer, 2007.

21. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill, 2004.

22. M.N. Mukherjee, Elements of Metric Spaces, Academic Publishers, 2005.

23. S. Kumaresan, Topology of Metric Spaces, 2nd Ed., Narosa Publishing House, 2011.

24. J. Sengupta, Metric Spaces, 4th Ed., U.N. Dhur & Sons Pvt. Ltd., 2005.

25. P.K. Jain and K. Ahmad, Metric Spaces, Narosa, 2004.

MTM-DC-MJ-504

LPP, Game Theory & Integral Transforms I
Credit: 4
Full Marks: 75 (CA: 25, SE: 50)

Learning Objectives:
The fundamental goal of this course is to introduce students to the core concepts and methodologies of Linear Programming, game theory and Laplace transform, including their properties, theoretical understanding and solution techniques.

Learning Outcomes:
On completion of the course, the students would

1. Understand and apply the properties of convex sets and convex functions in the context of LPPs, recognizing their significance for optimality.

2. Identify and interpret Basic Feasible Solutions as extreme points of the feasible region, and understand the relationships between adjacent extreme points.

3. Formulate and interpret the dual of an LPP, understanding the primal-dual relationships and their economic implications.

4. Solve Transportation Problems, Assignment Problem, Traveling Salesmen problem using various methods and determine optimal strategies for two-person zero-sum games, employing both graphical techniques and linear programming formulations.

5. Able to use fundamental theorems like the Convolution Theorem and the Heaviside's Expansion Theorem to efficiently find inverse Laplace transforms.
6. Apply the Laplace transform to solve ODEs, PDEs and evaluate definite integrations.

Course Contents

Module-1
Convex Sets and Convex Functions, Convex Hulls, Hyperplanes, Convex Polyhedral Sets and Polyhedral Cones, Simplexes, Extreme Points, Optimality and Unboundedness. Fundamental theorem of LPP. Simplex method, Theory of simplex method, Basic Feasible Solution, Optimality and unboundedness, Simplex algorithm, Artificial variables, Two-phase method, Big-M method, Degeneracy.

Module-2
Duality in LPP, Duality theorems, Dual simplex method, Primal dual method. Transportation problem: Northwest-corner method, Least cost method and Vogel approximation method, UV method. Unbalanced transportation problem. Degeneracy. Assignment problems: Hungarian method. Travelling Salesman problem.

Module-3
Game theory: Rectangular games, Pure and Mixed strategy, Saddle point and its existence, Optimal strategy and value. Necessary and sufficient condition for optimality. Fundamental Theorem of rectangular games. Algebraic, Graphical and Dominance methods. Relation between theory of games and LPP.

Module-4
Laplace and inverse Laplace transforms, Existence of Laplace Transform, Properties of Laplace transform, Laplace transform of periodic and special functions, First shifting properties. Convolution: Convolution theorem, Properties of Convolution. Differentiation and integration of Laplace transforms. Inversion formula of Laplace transform: Partial fraction decomposition, Convolution theorem. Heaviside's expansion theorem, Second shifting properties, Initial and final value theorem, Finite Laplace transform and their properties, Application of Laplace transform: Evaluation of definite integrals, Solution of ODEs and PDEs.

Reference Books

1. G. Hadley, Linear Programming, Narosa, 2002.
2. M.S. Bazaraa, J.J. Jarvis and H.D. Sherali, Linear Programming and Network Flows, 2nd Ed., Wiley, 2004.
3. P.K. Dutta, Strategies and Games: Theory and Practice, MIT Press, 1999.
4. L.F. Fernandez and H.S. Bierman, Game Theory with Economic Applications, Addison Wesley, 1998.
5. R.D. Gibbons, Game Theory for Applied Economists, Princeton University Press, 1992.
6. F.S. Hillier and G.J. Lieberman, Introduction to Operations Research, 9th Ed., McGraw Hill, 2009.

7. H.A. Taha, Operations Research: An Introduction, 8th Ed., Prentice Hall India, 2006.
8. J. Matoušek and B. Gärtner, Understanding and Using Linear Programming, Springer, 2006.
9. W.J. Cook, W.H. Cunningham, W.R. Pulleyblank and A. Schrijver, Combinatorial Optimization, Wiley-Interscience, 1997.
10. D. Solow, Linear Programming: An Introduction to Finite Improvement Algorithms, Dover, 2014.
11. D. Bertsimas and J.N. Tsitsiklis, Introduction to Linear Optimization, Athena Scientific, 1997.
12. G. Strang, Linear Algebra and its Applications, Thomson, 2007.
13. C.H. Papadimitriou and K. Steiglitz, Combinatorial Optimization: Algorithms and Complexity, Dover, 1998.
14. A. Schrijver, Theory of Linear and Integer Programming, Wiley, 1998.
15. S.I. Gass, Linear Programming: Methods and Applications, Dover, 2010.
16. V. Chvátal, Linear Programming, F.H. Freeman, 1983.
17. R.J. Vanderbei, Linear Programming: Foundations and Extensions, Springer, 2020.
18. D.G. Luenberger and Y. Ye, Linear and Nonlinear Programming, Springer, 2008.
19. M.J. Osborne and A. Rubinstein, A Course in Game Theory, MIT Press, 1994.
20. R.B. Myerson, Game Theory: Analysis of Conflict, Harvard University Press, 1997.
21. D. Fudenberg and J. Tirole, Game Theory, Ane Books, 2005.
22. S.R. Chakravarty, M. Mitra and P. Sarkar, A Course in Cooperative Game Theory, Cambridge University Press, 2015.
23. M. Maschler, S. Zamir and E. Solan, Game Theory, Cambridge University Press, 2013.
24. S.D. Sharma, Operations Research: Theory Methods & Applications, Kedar Nath Ram Nath, 2020.
25. R. Courant and D. Hilbert, Methods of Mathematical Physics, Vol. I and II, Interscience Publishers, New York, 1953.
26. I.N. Sneddon, The Uses of Integral Transforms, McGraw-Hill Book Company, New York, 1972.
27. C.J. Tranter, Integral Transforms in Mathematical Physics, John Wiley & Sons, New York, 1951.
28. M.R. Spiegel, Laplace Transforms, McGraw Hill, 1965.
29. E.J. Watson, Laplace Transforms and Application, Van Nostland Reinhold Co. Ltd., 1981.
30. M.D. Raisinghania, Advanced Differential Equations, S. Chand Publishing, 2024.
31. E. Kreyszig, Advanced Engineering Mathematics, John Wiley & Sons, 2023.

SEMESTER VI

MTM-DC-MJ-601

Probability, Statistics & Vector Calculus
Credit: 4
Full Marks: 75 (CA: 25, SE: 50)

Learning Objectives:
The fundamental goal of this course is to introduce students to the foundational concepts of probability theory, statistical knowledge and vector calculus. This includes developing a strong understanding of random variables (both discrete and continuous), their distributions, key statistical measures like expectation and moments etc, Green's theorem, Stokes' theorem and Gauss' divergence theorem and their applications.

Learning Outcomes:
On completion of the course, the students would

1. Characterize and differentiate between various discrete and continuous random variables, effectively utilizing their respective probability mass functions, probability density functions, and cumulative distribution functions.
2. Compute and interpret key statistical measures such as mathematical expectation, variance, moments, and moment generating functions for both discrete and continuous random variables.
3. Quantify relationships between two random variables by calculating and interpreting covariance and the correlation coefficient.
4. Apply various estimation methods, including the method of moments and maximum likelihood estimation, to determine point estimates for population parameters.
5. Evaluate double and triple integrals by using change of variables, and parameterize curves and surfaces and evaluate line, surface, and volume integrals of scalar and vector valued functions.
6. Apply Green's Theorem, Gauss' Divergence Theorem, and Stokes' Theorem to solve problems involving circulation, flux, and relationships between different types of integrals.

Course Contents

Module-1
Real random variables (discrete and continuous), Cumulative distribution function, Probability mass/density functions, mathematical expectation, moments, moment generating function, characteristic function. Discrete distributions: Rectangular, Binomial, Poisson, Geometric, Negative

binomial. Continuous distributions: Uniform, Normal, Exponential, Gamma, Beta. Transformation of a random variable. Joint Distributions: Discrete and continuous random variables and their properties, Joint probability mass/density functions. Marginal and conditional distributions, expectation, product moments, covariance, correlation coefficient, independence of random variables, bivariate normal distribution, joint moment generating function, linear regression, Chebyshev's inequality.

Module-2

Convergence in Probability, Strong and weak law of large numbers. Central limit theorem, Random Samples. Estimation: Unbiasedness, consistency, method of moments and method of maximum likelihood estimation, Bayes' estimator, Distributions of sampling statistics, Estimation of regression coefficients, Interval estimation. Confidence intervals: General principle, For the mean of Normal population of known/unknown variance, For variance of Normal population.

Module-3

Testing of hypothesis: Null and alternative hypotheses, critical and acceptance regions, two types of error, Level of significance, Power of test, Most powerful test and Neyman-Pearson Fundamental Lemma, Likelihood-ratio tests, Tests for normal population parameters based on normal, t, Chi-square distribution.

Module-4

Triple product, review of differentiation and integration of vector valued function, gradient, divergence and curl. Curves and their parameterization, line integration of vector functions, circulation. Surface and volume integration. Green's theorem, Stokes' theorem and Gauss' divergence theorem and their applications.

Reference Books

1. I. Miller and M. Miller, John E. Freund's Mathematical Statistics with Applications, 7th Ed., Pearson, 2006.
2. S. Ross, Introduction to Probability Models, 9th Ed., Academic Press, 2007.
3. R.B. Ash, Basic Probability Theory, Dover Publications, 2008.
4. R.V. Hogg, J.W. McKean and A.T. Craig, Introduction to Mathematical Statistics, Pearson, 2007.
5. A.M. Mood, F.A. Graybill and D.C. Boes, Introduction to the Theory of Statistics, 3rd Ed., McGraw Hill, 2007.
6. A. Gupta, Groundwork of Mathematical Probability and Statistics, Academic Publisher, 2015.
7. W. Feller, An Introduction to Probability Theory and its Applications, Wiley, 1968.
8. A.P. Baisnab and M. Jas, Elements of Probability and Statistics, McGraw Hill, 1993.
9. V.K. Rohatgi, A.K.Md.E. Saleh, An Introduction to Probability and Statistics, Wiley, 2008.
10. A.A. Borovkov, Probability Theory, Springer, 2009.
11. J. Pitman, Probability, Springer, 1993.
12. S.C. Gupta and V.K. Kapoor, Fundamentals of Mathematical Statistics, Sultan Chand and Sons, 2020.

13. A.M. Gun, M.K. Gupta and B. Dasgupta, Fundamentals of Statistics, Vol. I and II, World Press, 2016.

14. J.E. Marsden and A.J. Tromba, Vector Calculus, W.H. Freeman, 1996.

15. T. Tao, Analysis II, Hindustan Book Agency, 2006.

16. M.R. Speigel, S. Lipschutz and D. Spellman, Schaum's outline: Vector Analysis, McGraw Hill, 2017.

17. C.E. Weatherburn, Elementary Vector Analysis: With Application to Geometry and Physics, CBS Ltd., 1926.

18. J.H. Hubbard and B.B. Hubbard, Vector Calculus, Linear Algebra and Differential forms: A Unified Approach, Pearson, 1998.

19. S.J. Colley and S. Cañez, Vector Calculus, Pearson, 2022.

20. S. Dineen, Multivariate Calculus and Geometry, Springer, 2014.

21. B.E. Blank and S.G. Krantz, Calculus Multivariable, Wiley, 2011.

MTM-DC-MJ-602

Complex Analysis
Credit: 4
Full Marks: 75 (CA: 25, SE: 50)

Learning Objectives:
The principal objective of this course is to teach students the fundamentals of complex analysis. They will learn to analyze complex functions, understand differentiability through the Cauchy-Riemann equations, and master complex integration using Cauchy's and the Residue Theorem. The course also trains students to classify singularities, compute residues, apply the Residue Theorem, Argument Principle, and Rouché's Theorem, and understand analytic continuation.

Learning Outcomes:
On completion of the course, the students would

1. Able to visualize complex numbers using the stereographic projection and analyze the properties of key complex functions like polynomial, exponential, and logarithmic functions.

2. Able to determine the differentiability of a complex function using the Cauchy-Riemann equations and identify analytic, entire, and harmonic functions and understand the concept of a conformal map and its application in geometric transformations, particularly with linear fractional transformations.

3. Able to apply fundamental theorems of complex integration, including Cauchy's Theorem and Integral Formula, to evaluate contour integrals.

4. Able to use of Taylor's and Laurent series to represent and analyze complex functions, including determining the radius of convergence.
5. Able to classify the singularities of complex functions (removable, poles, and essential) and apply the powerful Residue Theorem to evaluate complex integrals.
6. Able to apply advanced theorems like Liouville's Theorem, the Maximum Modulus Principle, the Argument Principle, and Rouché's Theorem to solve complex problems and prove key results in the field.

Course Contents

Module-1
Stereographic projection. Complex valued functions, Polynomial functions, Rational functions, Exponential, Trigonometric and Hyperbolic functions, Multivalued function, Logarithmic function, Branch of a logarithm, Linear fractional transformations (Möbius transformations). Limit and continuity of complex valued functions.

Module-2
Differentiability, Cauchy-Riemann equations, sufficient conditions for differentiability, Analytic functions, Entire functions, Harmonic functions and Harmonic conjugates. Zeros of analytic functions, Identity Theorem. Conformal maps.

Module-3
The complex integral (over piecewise C^1 curves), ML-inequality, Fundamental theorem of calculus for analytic functions, Cauchy's Theorem and Integral Formula, Power series representation of analytic functions, The difference between Real Analytic functions and Complex Analytic functions. Morera's Theorem, Goursat's Theorem, Cauchy's inequality, Liouville's Theorem, Fundamental Theorem of Algebra, Weierstrass Convergence Theorem, Maximum Modulus Principle, Schwarz Lemma, Index of a closed curve, Contour, Index of a contour, Simply connected domains, Cauchy's Theorem for simply connected domains.

Module-4
Convergence of series, Power series, Radius of convergence, Differentiation of power series, absolute and uniform convergence of a power series, Taylor's Series, Laurent series. Definitions and Classification of singularities of complex functions, Isolated singularities: Removable singularities, Poles and Essential singularities. Meromorphic functions, Casorati-Weierstrass Theorem, Residues, Residue Theorem and its applications to contour integrals, Applications of Argument Principle, Rouché's Theorem, Open mapping theorem, Analytic Continuation.

Reference Books

1. J.B. Conway, Functions of One Complex Variable, 2nd Ed., Narosa Publishing House,
2. J.E. Marsden and M.J. Hoffman, Basic Complex Analysis, 3rd Ed., W.H. Freeman, New York, 1999.
3. H.A. Priestley, Introduction to Complex Analysis, Oxford University Press, 2003.
4. D. Sarason, Complex Function Theory, Hindustan Book Agency, Delhi, 1994.
5. L.V. Ahlfors, Complex Analysis, McGraw-Hill, 1979.

6. W. Rudin, Real and Complex Analysis, McGraw-Hill Book Co., 1966.
7. E. Hille, Analytic Function Theory, Vol. I and II, Ginn & Co., 1959.
8. T.W. Gamelin, Complex Analysis, Springer, 2001.
9. J. Bak and D.J. Newman, Complex Analysis, Springer, 2010.
10. T. Needham, Visual Complex Analysis, Clarendon Press, 1998.
11. S. Ponnusamy, Foundations of Complex Analysis, Narosa, 2008.
12. A.F. Beardon, Complex Analysis, Dover, 2020.
13. E.M. Stein and R. Shakarchi, Complex Analysis, Princeton University Press, 2003.
14. R. Remmert, Theory of Complex Functions, Springer, 1991.
15. S. Lang, Complex Analysis, Springer, 1999.
16. C.A. Bernstein and R. Gay, Complex Variables: An Introduction, Springer, 1991.
17. D.G. Zill and P. Shanahan, Complex Analysis, Jones and Bartlett Publishers, 2013.
18. J.W. Brown and R.V. Churchill, Complex Variables and Applications, McGraw Hill, 2013.
19. S. Ponnusamy and H. Silverman, Complex Variables with Applications, Birkhäuser, 2006.
20. R.P. Boas, Invitation to Complex Analysis, The Mathematical Association of America, 2010.
21. D.C. Ullrich, Complex Made Simple, American Mathematical Society, 2008.
22. R.P. Agarwal, K. Perera and S. Pinelas, An Introduction to Complex Analysis, Springer, 2011.
23. H.S. Kasana, Complex Variables: Theories and Applications, Prentice Hall India, 2005.
24. A. Gupta, Principles of Complex Analysis, Academic Publishers, 2015.

MTM-DC-MJ-603

PDE, Integral Equations & Integral Transforms II
Credit: 4
Full Marks: 75 (CA: 25, SE: 50)

Learning Objectives:
This course aims to develop a strong foundational understanding of second-order linear PDEs, including their classification, canonical forms, and solution techniques. Students will learn analytical methods such as separation of variables, integral transforms, Green's functions, and method of characteristics. The course also covers the theory and solution of linear integral equations of Fredholm and Volterra types, including kernel-based approaches and decomposition methods. Students

will gain proficiency in Fourier and Z-transforms and their applications in solving PDEs, ODEs, and integral equations. Emphasis will be placed on both theoretical understanding and practical problem-solving skills.

Learning Outcomes:

On completion of the course, the students would

1. Able to classify second-order linear partial differential equations and transform them into canonical forms and solve Laplace, Heat, and Wave equations using separation of variables, Green's functions, and integral transform methods.
2. Able to formulate and solve Fredholm and Volterra integral equations of the first and second kinds.
3. Able to apply techniques such as degenerate kernels, successive approximations, and Adomian decomposition methods to solve integral equations.
4. Able to analyze and solve singular integral equations including Abel-type and generalized Abel's problems.
5. Able to use Fourier and Z-transforms to solve differential and integral equations and evaluate definite integrals.
6. Able to demonstrate understanding of convolution theorems, Parseval's relations, and their applications in mathematical modeling.

Course Contents

Module-1

Second order linear PDEs: Classification, Canonical forms, Linear PDEs with constant coefficients. Solutions to Laplace, Heat and Wave equations: Elementary solution, Separation of variables, Method of characteristics, Integral transforms, Green's functions methods.

Module-2

Linear integral equations of 1st and 2nd kinds-Fredholm and Volterra types. Relation between integral equations and initial, boundary value problems. Degenerate kernel: Homogeneous and Non-homogeneous integral equations, First, Second and Third Fredholm theorems and their applications, Fredholm alternative theorem and its applications.

Module-3

Successive approximations: Iterated kernels, Reciprocal kernels, Volterra's solution of Fredholm's integral equation. Successive substitution, Adomian decomposition method, Modified decomposition method, Series solutions. Singular integral equations: Abel's problem, Generalized Abel's integral equation.

Module-4

Fourier integral theorem, Fourier and inverse Fourier transforms, Fourier cosine and sine transforms and their inverse transforms, Fourier transform of generalized functions, Properties of Fourier, Fourier cosine and Fourier sine transforms. Convolution: Convolution theorems, properties of convolution, Parseval's relation, General Parseval's relation. Bessel's inequality, Application of

Fourier, Fourier cosine and Fourier sine transforms: Evaluation of definite integrals, Solution of ODEs, PDEs, Integral equations, Mathematical statistics. Z-transforms and their properties, Inverse Z-transforms and their properties.

Reference Books

1. I.N. Sneddon, Elements of Partial Differential Equations, Dover, 2006.
2. W.E. Williams, Partial Differential Equations, Clarendon Press, 1980.
3. F.H. Miller, Partial Differential Equations, Wiley, 1941.
4. I.G. Petrovsky (Translated by A. Shenitzer), Lectures on Partial Differential Equations, University Press, 1954.
5. A. Sommerfeld, Partial Differential Equations in Physics, Academic Press, 1967.
6. E. Zauderer, Partial Differential Equations of Applied Mathematics, Wiley, 2006.
7. H.F. Weinberger, A First Course in Partial Differential Equations, Blaisdell, 1965.
8. C.R. Chester, Techniques in Partial Differential Equations, McGraw Hill, 1971.
9. L.C. Evans, Partial Differential Equations, American Mathematical Society, 2014.
10. V. Vladimirov, Equations of Mathematical Physics. Dekker, 1971.
11. T. Myint-U and L. Debnath, Linear Partial Differential Equations for Scientists and Engineers, Springer, 2006.
12. T. Amarnath, An Elementary Course in Partial Differential Equations, Narosa Publishing House, 2006.
13. R. Courant and D. Hilbert, Methods of Mathematical Physics, Vol. I and II, Interscience Publishers, New York, 1953.
14. I.N. Sneddon, The Uses of Integral Transforms, McGraw-Hill Book Company, New York, 1972.
15. J.W. Miles, Integral Transforms in Applied Mathematics, Cambridge University Press, 2008.
16. R.N. Bracewell, The Fourier Transform and Its Applications, McGraw Hill, 1986.
17. C.J. Tranter, Integral Transforms in Mathematical Physics, John Wiley & Sons, New York, 1951.
18. I.N. Sneddon, The Fourier Transform, Dover Publications, New York, 1995.
19. L. Andrews and V.K. Shivamoggi, Integral Transforms for Engineers, SPIE Press, Bellingham, WA, 1999.
20. A. Wazwaz, A First Course in Integral Equations, World Scientific Publishing Co. Pvt. Ltd., 1997.
21. R.P. Kanwal, Linear Integral Equations: Theory and Techniques, Academic Press Inc., 1971.
22. D. Porter and D.S.G. Stirling, Integral Equations, Cambridge University Press, 1971.
23. W.V. Lovitt, Linear Integral Equations, Dover Publications, New York, 1950.

24. F.G. Tricomi, Integral Equations, Dover Publications, New York, 1985.

25. S.G. Mikhlin, Linear Integral Equations, Dover Publications, Dover, 2020.

MTM-DC-MJ-604

Higher Abstract Algebra
Credit: 4
Full Marks: 75 (CA: 25, SE: 50)

Learning Objectives:
This course aims to provide an in-depth understanding of advanced group theory and ring theory concepts. Students will explore automorphisms, group actions, Sylow theorems, and classification of finite groups. They will study structural properties such as characteristic subgroups, direct products, and solvable and nilpotent groups. The course also introduces foundational ring-theoretic concepts including polynomial rings, divisibility, factorization, and domain classifications like Euclidean and principal ideal domains. Emphasis is placed on developing abstract reasoning, proof techniques, and the ability to apply theoretical concepts to solve algebraic problems.

Learning Outcomes:
On completion of the course, the students would

1. Understand and determine automorphisms, inner automorphisms, and characterize automorphism groups of cyclic groups.
2. Apply concepts of characteristic subgroups, commutator subgroups, and direct products, and use the Fundamental Theorem of Finite Abelian Groups.
3. Analyze group actions, compute stabilizers and kernels, and apply class equations and permutation representations.
4. Able to prove and apply Cayley's and Sylow's theorems and classify finite groups based on order.
5. Examine the structure of simple, nilpotent, and solvable groups using normal and composition series.
6. Study divisibility and factorization properties in integral domains, Euclidean domains, and principal ideal domains and use reducibility and irreducibility tests to analyze polynomials over commutative rings.

Course Contents

Module-1
Automorphism, inner automorphism, automorphism groups, automorphism groups of finite and infinite cyclic groups, applications of factor groups to automorphism groups, Characteristic subgroups, Commutator subgroups and their properties. Properties of external direct products, the group $\mathbf{U}_n$ of units modulo n as an external direct product, internal direct products, Fundamental Theorem of finite abelian groups.

Module-2
Group actions, stabilizers and kernels, permutation representation associated with a given group action. Applications of group actions. Class equation and consequences. Cayley's theorem, Generalized Cayley's theorem. Index theorem. Groups acting on themselves by conjugation, conjugacy in S_n.

Module-3
p-groups, Cauchy's theorem, Sylow's theorems and consequences. Classification of Finite Groups of order pq, p^2q, p^2q^2 (p, q are primes) etc. Simple Groups, Simplicity of A_n for $n \geq 5$, Non-simplicity tests, Simplicity of Groups of order ≤ 60. Normal Series, Composition Series. Nilpotent Groups, Solvable Groups.

Module-4
Polynomial rings over commutative rings, division algorithm and consequences. Divisibility in integral domains, irreducible, primes. Factorization domains, Euclidean domains, principal ideal domains, unique factorization domains, Factorization of polynomials, reducibility tests, irreducibility tests, Eisenstein criterion.

Reference Books

1. J.B. Fraleigh, A First Course in Abstract Algebra, 7th Ed., Pearson, 2002.
2. M. Artin, Abstract Algebra, 2nd Ed., Pearson, 2011.
3. J.A. Gallian, Contemporary Abstract Algebra, 4th Ed., Narosa, 1999.
4. S. Lang, Introduction to Linear Algebra, 2nd Ed., Springer, 2005.
5. G. Strang, Linear Algebra and its Applications, Thomson, 2007.
6. S. Kumaresan, Linear Algebra: A Geometric Approach, Prentice Hall of India, 1999.
7. K. Hoffman and R.A. Kunze, Linear Algebra, 2nd Ed., Prentice Hall of India, 1971.
8. S.H. Friedberg, A.L. Insel and L.E. Spence, Linear Algebra, Prentice Hall of India, 2004.
9. D.S. Dummit and R.M. Foote, Abstract Algebra, 3rd Ed., Wiley & Sons, 2004.
10. J.R. Durbin, Modern Algebra: An Introduction, John Wiley & Sons, 2000.
11. D.A.R. Wallace, Groups, Rings and Fields, Springer, 1998.
12. D.S. Malik, J.M. Mordeson and M.K. Sen, Fundamentals of Abstract Algebra, McGraw Hill, 1996.
13. I.N. Herstein, Topics in Algebra, Wiley, India, 1975.